Calculus (in class) Test 1 2010 Name KEY

Solve each of the inequalities

#1 Solve the inequality, be sure answer is in **interval notation**.

$$\frac{(5x+3)}{(x-2)(x+6)} \geq 0$$

#1 $(-6, -3/5] \cup (2, \infty)$

ZERO $x = -3/5$

RESTRICTION $x = -6, x = 2$

$(-6, -3/5] \cup (2, \infty)$

(−) ONE (+) 0 (−) ONE (+)

−6 −3/5 2

#2 Solve the inequality, be sure answer is in **interval notation**.

$$\frac{k^2+2k-35}{k^2+8k+12} \leq 0 \qquad \frac{(k+7)(k-5)}{(k+6)(k+2)} \leq 0$$

#2 $[-7, -6) \cup (-2, 5]$

ZERO $x = -7, x = 5$

RESTRICTION $x = -6, x = -2$

(+) 0 (−) ONE (+) ONE (−) 0 (+)

−7 −6 −2 5

$[-7, -6) \cup (-2, 5]$

Functions for problems 3 through 5

f(x) = $2x^2 + x - 3$ **g(x) = $-5x^2 + 3x - 7$** **h(x) = $4x + 1$**

#3 Find and simplify g (f (h (2))) $h(2) = 9$, $f(9) = 168$, $g(168) = -140{,}623$

#4 −140,623

#4 Find and simplify f(g(−5)) $g(-5) = -147$ $f(-147) = 43{,}068$

#5 43,068

#5 Find and simplify h(g (h(−2))) $h(-2) = -7$ $g(-7) = -273$ $h(-273) = -1091$

#6 −1091

Functions for problem #6

f(x) = $x - 3$, g(x) = $x^2 - x - 6$, h(x) = $x^2 + 9$

a. find f (5) + g (2) + h (0) $= 2 + (-4) + 9 = 7$

#6a 7

b. find f (f (3)) $f(3) = 0$ $f(0) = -3$

#6b −3

c. find (g (h (f (2))) $f(2) = -1$ $h(-1) = 10$ $g(10) = 84$

#6c 84

d. Find the **solution set** for f(g(x)) = g(f(x))

#6d $x = 2\frac{1}{2}$

$$[x^2-x-6]-3 = [x-3]^2-[x-3]-6$$
$$x^2-x-9 = x^2-6x+9-x+3-6$$
$$x^2-x-9 = x^2-7x+6$$
$$6x = 15$$
$$x = 2\tfrac{1}{2}$$

#7 Identify the **center** and the **radius**.

a) $x^2 + y^2 - 8x + 6y - 11 = 0$

$x^2 - 8x \quad + y^2 + 6y \quad = 11$

$(x-4)^2 + (y+3)^2 = 11 + 16 + 9$

b) $x^2 + y^2 + 12x - 5y - 6\frac{3}{4} = 0$

$x^2 + 12x \quad + y^2 - 5y \quad = 6\frac{3}{4}$

$(x+6)^2 \quad + (y - \frac{5}{2})^2 \quad = 6\frac{3}{4} + 36 + \frac{25}{4}$

#7a $\underline{\text{CENTER: } (4, -3) \quad \text{RADIUS: } 6}$

$(x-4)^2 + (y+3)^2 = 36$

#7b $\underline{\text{CENTER: } (-6, 2\frac{1}{2}) \quad \text{RADIUS: } 7}$

$(x+6)^2 + (y - \frac{5}{2})^2 = 49$

#8 Solve the following systems of equations

a. $\begin{aligned} 12x - y + 12z &= 6 \\ 2x + y - 2z &= -4 \\ 9x + 2y + 3z &= 3 \end{aligned}$

b. $\begin{aligned} 5x + 9y &= 19 \\ 2x - y &= -20 \end{aligned}$

c. $\begin{aligned} 2x - y + z &= 4 \\ x + y - z &= 11 \\ 4x - 2y + 2z &= 5 \end{aligned}$

a $\underline{(-2, 6, 3)}$ b $\underline{(-7, 6)}$ c $\underline{\text{NO SOLUTION}}$

d. $\begin{aligned} 3x + 6y - 6z &= 9 \\ 2x - 5y + 4z &= 6 \\ -x + 16y + 14z &= -3 \end{aligned}$

e. $\begin{aligned} 3x - 5y &= 1 \\ 2x + y &= -2 \end{aligned}$

f. $\begin{aligned} x + 3y - 2z &= 4 \\ 4x - y + z &= -1 \\ 3x - 4y + 3z &= -5 \end{aligned}$

d $\underline{(3, 0, 0)}$ e $\underline{(-\frac{9}{13}, -\frac{8}{13})}$ f $\underline{\text{NO SOLUTION}}$

#9 Write the equation of each circle

a. (-4 , 3)
(2 , 3)
(5 , 0)

b. (10 , 3)
(6 , -5)
(-2 , -1)

a $\underline{(x+1)^2 + (y+3)^2 = 45}$

$16 + 9 - 4A + 3B + C = 0$

$4 + 9 + 2A + 3B + C = 0$

$25 + 0 + 5A + 0B + C = 0$

$A = 2 \quad B = 6 \quad C = -35$

$x^2 + 2x \quad + y^2 + 6y \quad = 35$

$(x+1)^2 + (y+3)^2 = 45$

b $\underline{(x-4)^2 + (y-1)^2 = 40}$

$100 + 9 + 10A + 3B + C = 0$

$36 + 25 + 6A - 5B + C = 0$

$4 + 1 - 2A - B + C = 0$

$A = -8 \quad B = -2 \quad C = -23$

$x^2 - 8x \quad + y^2 - 2y \quad = 23$

$(x-4)^2 + (y-1)^2 = 40$

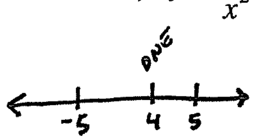
10 Graph and state the domain and range using **interval notation.**

a) $y = |3(x-2)^2 - 7|$

10a Domain: $(-\infty, \infty)$

10a Range: $[0, \infty)$

b) $y = \dfrac{x^2 - 25}{x^2 - 8x + 16} = \dfrac{(x-5)(x+5)}{(x-4)^2}$



10b Domain: $(-\infty, 4) \cup (4, \infty)$

10b Range: $(-\infty, 27/9]$

*The **maximum height** or greatest **y–value** of the function is not the same as the **extreme behavior** of the function. Use a graphing utility to explore this function. When the graph of the function is on the screen use the 2nd Calc option, select maximum, then select appropriate left and right bounds to isolate the suspected maximum. Finally the calculator will prompt for a guess within the designated boundaries and report the maximum value for the function given the domain it was to search under.

#11 Write each of the following in standard H , K forms identify characteristics of each

a. $x^2 + y^2 - 12x + 6y = 19$

$x^2 - 12x \quad + y^2 + 6y = 19$

$(x-6)^2 + (y+3)^2 = 64$

#a $(x-6)^2 + (y+3)^2 = 64$

Characteristics:

CENTER $(6,-3)$ RADIUS 8

b. $4x^2 + y^2 - 32x + 16y + 124 = 0$

$4[x^2 - 8x \quad] + y^2 + 16y \quad = -124$

$4(x-4)^2 + (y+8)^2 = 4$

$\dfrac{(x-4)^2}{1} + \dfrac{(y+8)^2}{4} = 1$

#a $\dfrac{(x-4)^2}{1} + \dfrac{(y+8)^2}{4} = 1$

Characteristics:

CENTER $(4,-8)$

VERTICAL / MAJOR AXIS LENGTH 4

MINOR AXIS LENGTH 2

c. $x^2 + 2x - y + 3 = 0$

$x^2 + 2x \quad + 3 = y$

$x^2 + 2x + 1 \quad + 2 = y$

$y = (x+1)^2 + 2$

#a $y = (x+1)^2 + 2$

Characteristics:

VERTEX $(-1, 2)$

STANDARD OPENING

OPENS UPWARD

#12 Solve

a) $2|x+3|^2 - 13|x+3| = -15$

$2m^2 - 13m + 15 = 0$ WHERE $m = |x+3|$

$(2m-3)(m-5) = 0$

⋮

$x = \{-8, -4\frac{1}{2}, -1\frac{1}{2}, 2\}$

#12a $x = \{-8, -4\frac{1}{2}, -1\frac{1}{2}, 2\}$

b) $2|4x-3|^2 + 17|4x-3| = -21$

#12b $x = \{\}$

EMPTY SET

c) $3|2x-7|^2 - 5|2x-7| = 2$

$x = 2\frac{1}{2}, 4\frac{1}{2}$

#12c $x = \{2\frac{1}{2}, 4\frac{1}{2}\}$

* Answers need to be written as fractions or mixed numbers. However, if one wanted to check answers by graphing the function that would be acceptable. Procedure – set each equation equal to zero, in other words move the constant from the right to the left hand side using the appropriate operation of addition or subtraction. Next graph the expression using the equation editor on the calculator. Finally, use the 2nd calc option and select zero. Then follow the on–screen instructions to find the zeros. Those results should match the solutions arrived at above.

#13 Find the equation of the circle in h, k form where the distance between (x , y) and (6 , -5) is $\sqrt{2}$ times the distance between (x , y) and (2 , -1).

$(x-6)^2 + (y+5)^2 = 2\left[(x-2)^2 + (y+1)^2\right]$

#13 $(x+2)^2 + (y-3)^2 = 64$

$x^2 - 12x + 36 + y^2 + 10y + 25 = 2x^2 - 8x + 8 + 2y^2 + 4y + 2$

$0 = x^2 + 4x - 28 + y^2 - 6y - 23$

$51 + 13 = x^2 + 4x + 4 + y^2 - 6y + 9$

$64 = (x+2)^2 + (y-3)^2$

#14. Find the area of of the triangle with vertices at (-8 , 5), (-3 , -2), and (1 , 4)

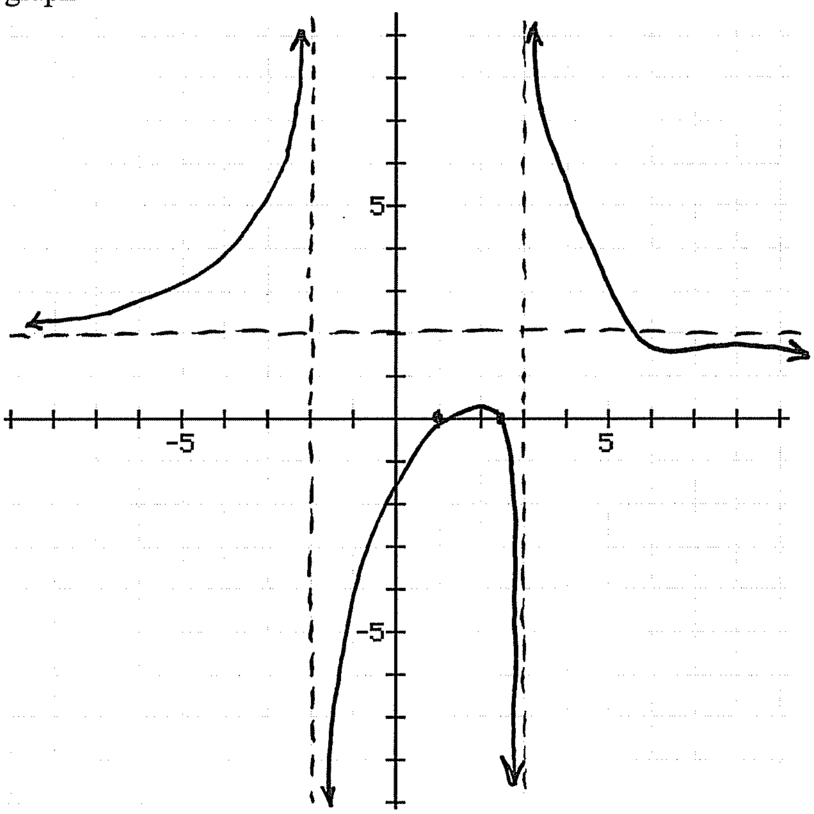
$$\frac{1}{2} \text{ DET } \begin{vmatrix} -8 & 5 & 1 \\ -3 & -2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 29$$

#14 29 units^2

Graph and identify zeroes, all asypmtotes, and perform a sign check, remember to check extreme values.

15. $y = \dfrac{2x^2 - 7x + 5}{x^2 - x - 6} = \dfrac{(2x-5)(x-1)}{(x-3)(x+2)}$

graph



zeroes $x = 1 \quad x = 2\tfrac{1}{2}$

vertical asymptotes $x = -2 \quad x = 3$

horizontal asymptotes $y = 2$

sign check

(+) ONE (−) 0 (+) 0 (−) ONE (+)

← −2 — 1 — $2\tfrac{1}{2}$ — 3 →

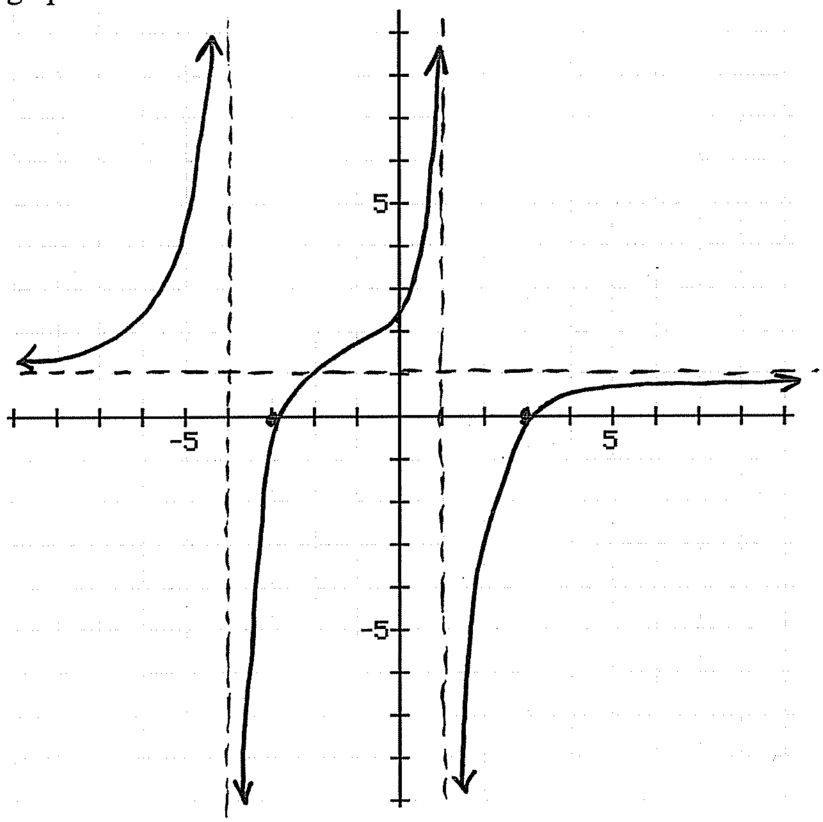
$\lim_{x \to -\infty} f(x) = 2^+$ $\lim_{x \to \infty} f(x) = 2^-$

*Remember that extreme behavior can be checked by selecting y_1 from the var button.

Graph and identify zeroes, all asypmtotes, and perform a sign check.

16. $\dfrac{x^2-9}{x^2+3x-4} = \dfrac{(x-3)(x+3)}{(x+4)(x-1)}$

graph



zeroes $x=-3$, $x=3$

vertical asymptotes $x=-4$ $x=1$

horizontal asymptotes $y=1$

sign check

(+)	UND	(−)	0	(+)	UND	(−)	0	(+)
	−4		−3		1		3	

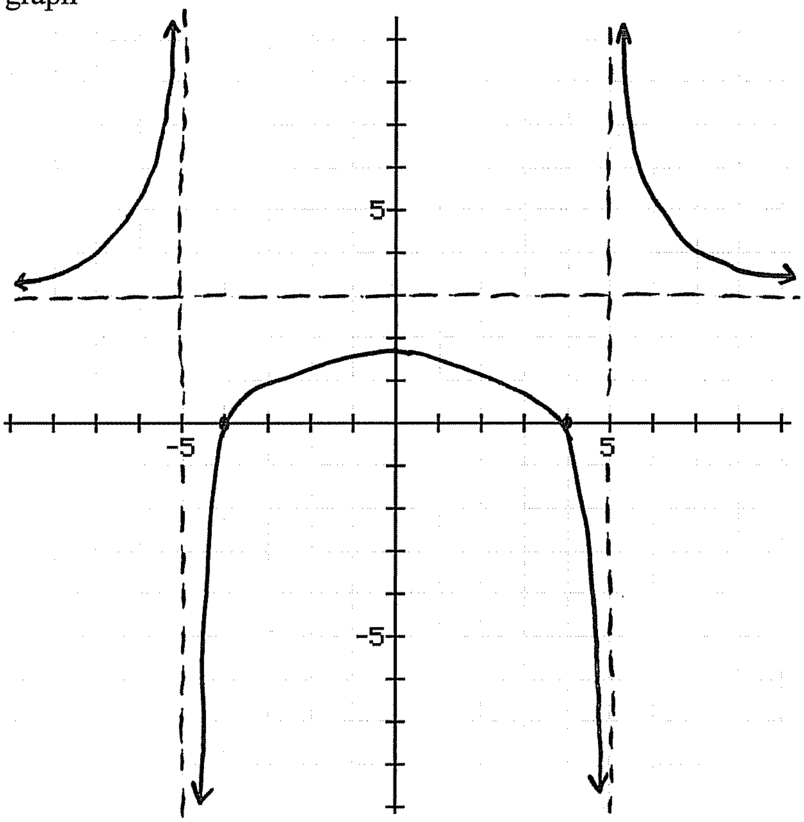
$\lim_{x\to -\infty} f(x) = 1^+$ $\lim_{x\to \infty} f(x) = 1^-$

*Remember that extreme behavior can be checked by selecting y_1 from the var button.

Graph and identify zeroes, all asypmtotes, and perform a sign check.

17. $\dfrac{3x^2-48}{x^2-25} = \dfrac{3(x-4)(x+4)}{(x-5)(x+5)}$

graph



zeroes $x=-4 \quad x=4$

vertical asymptotes $x=-5 \quad x=5$

horizontal asymptotes $y=3$

sign check

(+)	UND	(−)	0	(+)	0	(−)	UND	(+)
	-5		-4		4		5	

$\lim\limits_{x\to-\infty} f(x) = 3^+$

$\lim\limits_{x\to\infty} f(x) = 3^+$